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Basic Mathematical Investigations in Electromagnetic Wave Theory

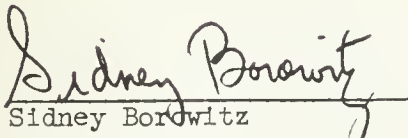
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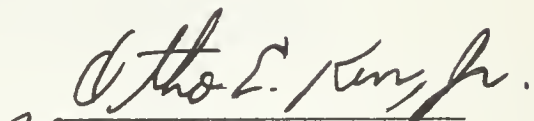
MARCH, 1959

NEW YORK UNIVERSITY
Institute of Mathematical Sciences
Division of Electromagnetic Research

FINAL REPORT
under
Contract No. AF 19(604)1717, March, 1959

BASIC MATHEMATICAL INVESTIGATIONS
IN ELECTROMAGNETIC WAVE THEORY


Sidney Borowitz
Acting Director


for Dr. Werner Gerbes
Contract Monitor

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1. Introduction

During the period from September 1, 1956, to March 31, 1959, the Division of Electromagnetic Research of the Institute of Mathematical Sciences at New York University has pursued a research program in electromagnetic theory under Contract Number AF 19(604)1717 with the Air Force Cambridge Research Center. This program has been a continuation of work done under Contract Number AF 19(122)-42 with the same agency. Not all of the fields of investigation permitted under the contract have been pursued. Individual scientists have chosen their own specific areas of interest in accordance with the broad objectives of the program outlined in the contract.

The work, which has been completed, falls into several categories. The first is problems of diffraction, in which the asymptotic solution of a variety of diffraction problems have been investigated. In some cases, the asymptotic form of a solution of a given problem has been exhibited and compared with the geometrical optics solution developed by Professor Keller (EM-109 and -121); in other cases, interesting applications of the geometrical optics solution have been given (Keller and Levy, EM-102; Keller, EM-104, -127, -119 and -110). Another aspect of the work on diffraction has been to consider problems in which modified surfaces are involved. In EM-108 (Karp) and -105 (Radlow), the field of a unidirectional conducting plane excited by a plane wave and by a dipole, respectively, are investigated. In EM-129 (Karp), the case of the diffraction of a scalar wave by a right-angled wedge with a partially absorbing surface is considered. EM-111 (Karal and Karp) treats the same problem for a vector wave.

Other work in diffraction includes EM-126 (Zitron and Karp) which gives a technique for exhibiting the solution of two and three dimensional multiple scattering problems in inverse powers of $d^{1/2}$, (d is the obstacle spacing), in terms of the solution of a single scattering problem.

In EM-120 (Felsen and Karp), an equivalence relation is found between the solutions of three dimensional electromagnetic problems for axially symmetric configurations supported by a thin fin and the two dimensional profile of this geometry.

Another general field of investigation has been surface waves. In EM-99, Friedman and Williams consider the problem of how to place a dipole above a dielectric so that a maximum amount of energy is carried by the surface wave (or that the surface wave be as pure as possible). EM-116, -123 and -124 (Karp and Karal) constitute part of a general investigation of the effect of wedge edges on the excitation efficiency of surface waves. Report EM-116 considers a right-angled wedge, both of whose faces can support surface waves excited by a dipole on the surface; in EM-124, the same problem is considered for a wedge, only one of whose surfaces can support surface waves. In EM-123, the diffraction of plane waves by a right-angled wedge, one of whose surfaces can support surface waves, is solved.

EM-128 (Keller and Karal) applies the theory of geometrical optics, developed for diffraction problems, to the study of the excitation and propagation of surface waves.

Another large area of investigation has been the statistical problems involved in radio wave propagation and in scattering, as well as in questions of optics. In Report EM-98 (Silverman), the mathematical

problems associated with the scattering of radio waves by refractive index fluctuations is considered; in EM-101 (Silverman), the problem of the short period fading of radio waves scattered by dielectric turbulence is analyzed. In EM-106 (Wolf), the coherence properties of optical wave trains is analyzed in relationship to their frequency characteristics.

In the course of investigating problems in electromagnetics, related mathematical questions, which have some intrinsic interest, always arise. In EM-100 (Jones and Kline), a method for dealing with the asymptotic evaluation of two dimensional integrals of the form $\int_D \int g(x,y) e^{ikf(x,y)} dy dx$ for large k is given. In EM-107 (Ragab), the inverse Laplace transform of some exponential functions is derived and, in EM-118 (Radlow), the double Laplace transforms of some Green's functions, of interest in electromagnetic theory, are exhibited.

There have also been some miscellaneous reports issued on this contract. One, EM-103 (Shenitzer), is a translation of three Russian papers on diffraction theory. Another, EM-114 (Moses), is a formulation of Maxwell's equations in spinor notation as well as a solution of the problem of determining the source distribution which would produce a given radiation pattern.

During the contract period, a total of 23 reports have been issued, 17 of which have been published or submitted for publication in technical journals.

We take pride in the fact that a portion of our contract activity can be devoted to the development of new talent in this important field. During the contract period, the following students obtained their doctorates and were, either wholly or in part, supported by contract funds:

James Radlow (June, 1957) Now with Grumman Aircraft
Willard Miranker (June, 1957) Now with I.B.M.
Bernard Seckler (February, 1958) Now teaching at
Pratt Institute, Brooklyn, N.Y.
Robert Lewis (February, 1958) Currently at N.Y.U.,
Institute of Mathematical Sciences
Bertram Levy (February, 1958) Institute of Math-
ematical Sciences
Eugene Pflumm (February, 1959) Institute of Math-
ematical Sciences
Norman Zitron (February, 1959) Currently at
Harvard University

We have also been fortunate in having a number of guests from other institutions visit us during this report period. The following are people whose visits were made possible by contract funds:

Emil Wolf (Jan. 1, 1957 - Feb. 7, 1958)
- University of Manchester, England
F.M. Ragab (Jan. 15, 1957 - Aug. 1, 1957)
- Eins Shans University, Cairo, Egypt
Claus Müller (Feb. 15, 1957 - May 1, 1957)
(Feb. 15, 1958 - Apr. 30, 1958)
- Technische Hochschule, Aachen, Germany
Joseph Meixner (Aug. 15, 1957 - Oct. 15, 1957)
- Technische Hochschule, Aachen, Germany
Ben Noble (Jan. 12, 1959 - Apr. 1, 1959)
- Royal College of Science and
Technology, Glasgow
Rolf Leis (Sept. 17, 1958 - April 1, 1958)
- Technische Hochschule, Aachen, Germany
Marcel Riesz (Feb. 2 - 14, 1959)
- University of Maryland

We shall now describe in somewhat more detail the work that has been completed on the contract. (A complete list of reports will be found on pps.20-23 of this report. Bibliographical information is contained in individual reports.)

2. Diffraction Theory

We include under this heading work done on diffraction problems involving short wavelength radiation, diffraction problems involving modified surfaces and some miscellaneous questions.

A. Diffraction Problems Involving Short Wavelength Radiation

During the tenure of the previous contract a method was developed for extending the theory of geometrical optics to include diffraction effects, and was applied to a variety of problems for which exact solutions were known. The asymptotic form of the exact solutions in each case corresponded to the solution obtained by this method. In EM-115 (Keller), this geometrical theory of diffraction has now been set down in its clearest form. This method of dealing with diffraction of short wavelength radiation is derived on the basis of physical rather than rigorous mathematical arguments and it is felt that to justify the method, further comparison should be made to as many problems as possible. To that end, the diffraction of plane waves by an elliptic cylinder (EM-121, Levy) and by a smooth convex obstacle (EM-109, Keller and Levy) is solved by standard separation of variables techniques and the asymptotic form of these solutions compared with the results obtained by the geometrical optics technique developed by Professor Keller. In both of these cases the method is found to be completely justified.

Since use of the geometrical optics method seems so well justified, it has been applied to a number of problems for which exact solutions are not available. In EM-102 (Keller and Levy), it is used to deduce the shape of a pulse diffracted around the earth and, in EM-104 (Keller), to

calculate the acoustic forces and torques on discs. Some previous results due to H. Levine are confirmed in this report and many new ones obtained. In EM-119 (Keller), this method is used to determine the effect of the thickness of a screen on the darkness of its shadow and, in EM-127 (Keller), to calculate the back scattering from a finite cone. Finally, in EM-110 (Keller), the method reveals a novel application in a so-called inverse problem - namely, finding the shape of an object from the pattern of the field which it scatters. Of course, the result obtains only in the short wavelength limit, but for high frequency radiation it enables one to design a reflector to produce a desired scattered field pattern.

B. Unidirectional Conducting Screens

A recent experiment has succeeded in measuring the torque supplied to a screen consisting of parallel fine wires embedded in a thin plastic sheet. The theory of this experiment was given by Toraldo di Francia, who idealized the screen as an infinitely thin anisotropic conducting screen, and who solved the problem in the limit $ka \rightarrow 0$, where a is the radius of the disc and $k = 2\pi/\lambda$ with λ as the wavelength. Since the work of Toraldo obtains in the limit of long wavelength, it is not strictly applicable to discs of moderate size and he suggested, during a visit to New York University, that the problem of the diffraction of a semi-infinite screen with anisotropic conductivity be attempted although he could not solve the problem himself. Such a screen would serve as an approximation to a large circular screen coplanar and tangent to the disc. This problem has been solved in EM-108 (Karp) for plane wave excitation and, in EM-105 (Radlow), for dipole excitation. In the course of finding the solution, new methods have been

developed for solving the conducting half plane problem and for determining the far fields. One introduces a function v related to the solution u of our problem by

$$\frac{\partial u}{\partial x} - i k \cos \theta_0 u = v ,$$

where θ is the angle of incidence of the wave. The effect of this transformation is to remove the incident plane wave. The boundary conditions on v are always at least as simple as those imposed on u and, thus, one can solve the problem in terms of the function v . The far field of u can be found from the far field of v by algebraic manipulation. Techniques similar to this have been used in solving some problems involving surface waves.

One other result of the original experiment is the charring of the plastic along the rim. This was correlated by Toraldo qualitatively with the edge behavior of the field along the rim. In EM-108 the near field is determined and the experimental results confirmed.

By reciprocity, the near field of the plane wave excitation can yield the far field with dipole excitation. In EM-105 the dipole excitation problem is treated. It is attacked by Wiener-Hopf techniques; a system of transform equations is obtained which can be uncoupled. An additional feature of interest is the fact that the transforms of the field quantities involve two complex variables. By carefully restricting the range of one of the variables, it can be regarded as a parameter, and the analysis can be carried out successfully.

C. Diffraction by Absorbers

There has been a great deal of interest recently in diffraction effects in the case of obstacles coated with absorbers. We investigate this problem in EM-129 and -108 (Karp). The work concerns diffraction by right angled wedges and exhibits a simple example of an easy technique for treating more complicated problems. The problem can be formulated as follows: on the absorbing surface an impedance boundary condition is imposed (i.e., a linear relation between the tangential components of $\vec{E}$ and $\vec{H}$). The problem can be solved in the case of an infinite plane surface with the result that there is perfect absorption at a special angle of incidence of a plane wave. However, when the surface is not plane, (i.e., if there is a corner or if it is wedge-like) the problem is difficult to solve since it is not separable. A technique previously used in water wave theory is adopted to overcome this difficulty. It is found that the absorbing wedge solution can be expressed by quadratures in terms of the conducting wedge solution. The far field technique of EM-108 is elaborated and it is found that, as far as the far field of the absorber is concerned, it can be deduced by purely algebraic operations on the solution of the far field of the conductor. In EM-129 the scalar two-dimensional line source problem is treated and, in EM-111 (Karp and Karal), the more complicated vectorial problem resulting when a skew plane wave excites the wedge is examined.

The work, summarized here, is continuing. It is of classical interest because it supplies a boundary value treatment for a 'black' screen.

D. Multiple Scattering

A previous investigation brought to light a technique for solving the two dimensional multiple scattering problem in the limit of large spacing of obstacles. A summary of this method was given in the last Final Report, EM-97. It was found that the first correction to single scattering required only the knowledge of the single scattered complex amplitudes themselves. In EM-126 (Zitron and Karp) it is shown that in both two and three dimensional problems all higher order corrections can be calculated with no additional information. Of course, the calculations become progressively more tedious. If d is the spacing between two cylinders of arbitrary shape, then in Part I of EM-126 the higher order correction of order $d^{-1/2}$, d^{-1} and $d^{-3/2}$ is calculated explicitly. In EM-126, Part II, corresponding calculations are performed in the scalar three dimensional case. This work is being extended to cover the vector case. A general expansion theorem for radiated fields, valid in a region of fixed size which is at a large distance from a scatterer, is obtained but has not yet been reported. The results obtained so far allow general investigations without going into irrelevant details, as shown in EM-85 (Karp) and EM-90 (Karp and Radlow). It should prove useful in quantum mechanics as well, once it is appreciated that multiple scattering can be expressed explicitly in terms of the scattering matrix (θ, θ_0) of the single scatterer.

E. An Equivalence Relation Between Certain Two and Three
Dimensional Geometries

In EM-120 (Karp and Felsen) a relation is exhibited between electromagnetic problems involving axially symmetric objects supported by a thin conducting fin and the two dimensional problem of its profile. Thus, a ring source, cone or sphere supported by a fin is related to a line source over a plane, a semi-circular boss in a plane and a wedge and a plane. The relation between the solutions is

$$u(r, z, \theta) = \frac{1}{\sqrt{r}} e^{\pm i \phi/2} F(r, z),$$

where u is the solution of the three dimensional problem

$$\nabla^2 u + k^2 u = 0$$

and F is the solution of

$$F_{rr} + F_{zz} + k^2 F = 0.$$

The solution of the three dimensional problem is expressed in terms of the corresponding scalar acoustical problems and a coupling term. The coupling term is derived by quadratures in terms of the variation of the edge singularity of the two dimensional problem. This method is then applied to obtain an explicit solution for the diffraction of an arbitrary electromagnetic wave by a half plane.

The results obtained here were begun under the previous contract and were reported there under the general heading of Edge Behavior in Diffraction Theory. This report is being issued jointly with the Brooklyn Polytechnic Institute. Professor L. Felsen of that institution is co-author.

3. Surface Wave Phenomena

The techniques described under heading C above, if appropriately modified at an early stage, can be used to treat the diffraction and excitation problem also in the case in which the modification of the boundary is of the type to allow surface waves. In EM-116 (Karp and Karal) the infinite plane problem is treated in the special case in which the dipole is on the surface. Then the effect of a corner can be investigated. Thus a modified surface in the form of a right angled corner can be treated - the source being at the vertex - and the amplitude of the surface waves generated is determined and compared with the effect in the plane case. In EM-116 both faces of the wedge can support surface waves. In EM-124 (Karp and Karal) the same problem is treated for the case in which only one face can support surface waves. From symmetry considerations, this latter problem can be regarded as a case in which a free space of angle 3π is available as compared with a free space of angle $3\pi/2$, as in EM-116. This work is part of a general investigation which aims (in part) at determining the effect of the wedge angle on excitation efficiency. It is also part of a mathematical program aimed at developing an algebraic technique for solving canonical diffraction problems of the type which are required as a basis for the application of geometrical diffraction theory.

In EM-123 (Karp and Karal) diffraction of a plane wave by a right angled wedge which sustains surface waves on one face is treated. The techniques are similar to those used in EM-124. The information obtained is expected to form part of a general scheme and comparison of the fields of sources on the body with fields due to diffraction will shed light on the problem of reciprocity. The interesting feature of EM-123 is the fact that part of the energy scattered by the tip of the wedge is sent out along the surface as a surface wave rather than being radiated out as a cylindrical wave.

In EM-99 (Friedman and Williams) the problem of the optimal method of excitation of surface waves by a dipole over a dielectric is treated. This is a generalization of the work of Cullen.

Another investigation of the excitation and propagation of surface waves is the application of the geometrical theory to this problem (EM-128, Keller and Karal). In this paper complex rays are introduced. As in the application of this method to diffraction theory, one determines an excitation coefficient by solving a canonical problem - in this case the solution of a line source over an impedance plane. With this canonical solution one may now discuss the propagation over arbitrary surfaces with constant or variable properties.

The method is applied to the surface waves excited by a line source on a wedge with variable surface impedance and line above on a cylinder. In both cases the solution obtained here agrees with the asymptotic form of the exact solution.

4. Statistical Problems

A. Scattering by Refractive Index Fluctuations

Recently, scattering of radio waves by refractive index fluctuations, presumably of turbulent origin, has come into prominence as an important new mode of radio communication. Evidently, the dielectric turbulence always present in both the troposphere and ionosphere is strong enough to support scattered signals which can readily be detected at great distances beyond the horizon. (However, it should be noted that, at least in the case of the troposphere, the role of layers in trans-horizon propagation is still an open question.) Study of this 'scatter mode' of propagation has been pursued in several reports.

One report, EM-98 (Silverman), is devoted to making more precise the kind of mathematical questions encountered in the radio scattering problem. A unified formulation of scattering theory in the language of random functions is given and the statistical characteristics of dielectric turbulence are analyzed. It is shown that the amplitude of the received scattered signal is a zero-mean, complex random function of time, which is quite accurately Gaussian and, therefore, characterized by its second moments or covariance. The relation between this covariance and the appropriate statistical quantities describing the dielectric turbulence is then derived. Special attention is given to the fact that the random functions involved are not strictly stationary, but only approximately stationary in suitably restricted space and time domains, i.e., 'locally stationary'.

These considerations have led ultimately to a new theory of locally stationary random processes, described elsewhere^{1,2,3}

The received scattered signal is, of course, always characterized by both short-pitched and long-pitched fading. In another report, (EM-101)⁴, Silverman the origin of the short-pitched fading is analyzed under the assumption of very narrow antenna beams. The fading of radio waves scattered by dielectric turbulence is shown to be the result of two effects. The first is time variation of the scattering eddies as seen in a coordinate system moving with the local wind velocity. The second is Doppler shifting produced by the convection of the scattering eddies by the mean wind and by the macro-eddies. Under the assumption that in the troposphere the scattering eddies lie in the inertial range of statistical turbulence theory, an expression is derived for the envelope fading rate of the received scattered signal as a function of carrier frequency. The result obtained is significantly different from that found with 'scattering blob' models. (In a later paper⁵ the effects of broad beam antennas and wind shear in the scattering volume are included.)

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1. R.A. Silverman, Locally stationary random processes, IRE Trans. Info. Th., IT-3, 182 (1957).
 2. R.A. Silverman, Scattering of plane waves by locally homogeneous dielectric noise, Proc. Camb. Phil. Soc., 52, 530 (1958).
 3. R.A. Silverman, A matching theorem for locally stationary random processes, Comm. Pure Appl. Math., in press (1959).
 4. R.A. Silverman, Fading of radio waves scattered by dielectric turbulence, J. Appl. Phys., 28, 506 (1957).
 5. R.A. Silverman, Remarks on the fading of scattered radio waves, IRE Trans. Antennas Prop., AP-6, 378 (1958).

A problem of random process theory, originally suggested by a question arising in radio scattering and discussed heuristically in RM-98, is dealt with in another report, (RM-125, Shapiro and Silverman). In many applications involving random noise, although the object of primary interest is a stationary random process, $x(t)$ say, what is actually available for observation instead is a weighted random process $y(t) = w(t)x(t)$, obtained by multiplying $x(t)$ by some non-random weight function $w(t)$. For example, in radio scattering, the weight function arises as a result of the joint effect of the gain patterns of the transmitting and receiving antennas. The question arises as to the relation between the power spectra of $x(t)$ and $y(t)$; this leads to an interesting mathematical problem concerning the asymptotic behavior of correlations. It is found that if the power spectrum of the process $x(t)$ is suitably well-behaved and properly matched to the weight function, then the high-frequency behavior of the power spectrum of $y(t)$, and more generally of its spectral covariance, is especially simple. Asymptotic theorems describing this behavior precisely are given.

B. Coherence Theory of Optics

A problem of considerable interest in optics is to relate the coherence properties of optical wave trains to their frequency characteristics. What is needed is an analog of the uncertainty relation of quantum mechanics, suitably modified and recast in an optical context. The optical case differs from the quantum mechanical case in two respects: (1) Optical wave functions are not complex, but intrinsically real. (2) Optical wave trains are to be thought of as realizations of a random process, so that a correlation theory description is appropriate. These questions are dealt with in a recent report, (EM-106¹, Wolf). A reciprocity relation, patterned after the uncertainty relation of quantum mechanics, is developed with due regard for the fact that the physically meaningful spectral bandwidth involves only positive frequencies.

A quantitative definition of the coherence time of an optical wave train is given and, then, related to the Michelson fringe visibility function, an experimentally measurable quantity. Generalizations to the case of two optical wave trains are developed. (Further results stemming from mathematical ideas initiated in this report have appeared elsewhere^{2,3}.)

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1. E. Wolf, Reciprocity inequalities, coherence time and bandwidth in signal analysis and optics, Proc. Roy. Soc., 71, 257 (1958).
 2. I. Kay and R.A. Silverman, On the uncertainty relation for real signals, Info. and Control, 1, 64 (1957).
 3. R.A. Silverman, Moments of analytic signals, IRE Trans. Circuit Th., CT-5, 84 (1958).

5. Mathematical Results

In this section, we shall describe some miscellaneous mathematical results which have arisen in connection with electromagnetic problems. In EM-100 (Jones and Kline), the asymptotic evaluation of the two dimensional integral $J = \iint g(x,y) e^{ikf(x,y)} dx dy$, with $f(x,y)$ defined in the range $m \leq f(x,y) \leq M$, is considered for large k . The basic idea of the paper is to replace the function e^{ikf} by $\int_{m-\epsilon}^{M+\epsilon} e^{ikt} \delta(t-f) dt$. The discussion of the two dimensional integral is thus reduced to a discussion of the single Fourier integral $J = \int_{m-\epsilon}^{M+\epsilon} e^{ikt} h(t) dt$ where $h(t) = \iint_D g(x,y) \delta[t-f(x,y)] dx dy$ and this discussion can be successfully carried through.

In EM-107 (Ragab) the inverse Laplace transform of the function $p^{\mu-1} \exp(-a^{1/m} p^{1/m})$ is found. This is a transform which has not been previously calculated but which arises in the pulse solution of some problems in electromagnetic theory.

During the course of this contract, a considerable effort has been put into the problem of diffraction of the quarter plane. The method which has been tried is a generalization of the Wiener-Hopf technique involving the theory of functions of two complex variables. One of the results of these efforts is the development of a method of obtaining the double Laplace transforms of functions. In EM-118 (Radlow), this method is applied to obtaining the double Laplace transform of the Green's

function of a linear second order elliptic differential equation in two variables. As a particular example, the double Laplace transform of

$H_0^{(1,2)} \left(k \sqrt{x_1^2 + x_2^2} \right)$ is calculated.

6. Miscellaneous

During the period of this contract, there were three published reports which do not fit into any of the categories used above. The first, EM-103 (Shenitzer), is a translation from the Russian of three papers on electromagnetic theory. These are:

1. The Electromagnetic Field of a Line Source Located in the Interior of an Ideally Conducting Parabolic Screen -
G.A. Grinberg, N.N. Lebedev, I.P. Skalskaia and Ya.S. Uflyand
2. The Electromagnetic Field of a Dipole Located in the Interior of a Parabolic Reflector -
I.P. Skalskaia
3. Derivation of the Laws of Reflection of Geometrical Optics on the Basis of an Asymptotic Treatment of the Diffraction Problem -
M.I. Kontorovitch and U.K. Muravev

In EM-112 (Wolf), an integral representation is obtained for the electromagnetic field in the image space of an optical system. This is in contrast with the usual geometrical optics analysis which does not give an indication of the vectorial features of the image such as the state of polarization of the image. In addition, this method is capable of dealing with optical systems of wide aperture.

Finally, in EM-114 (Moses), a formulation is given of Maxwell's equations in spinor notation so that its similarities and differences with the Dirac equation of the electron can be exhibited. Thus, it is now possible to apply

some previously derived results of the inverse problem in Dirac theory to the problem of determining a source distribution which will give a particular radiation pattern. In general, the source distribution is not found to be unique, but it is possible to make conditions which will specify them completely.

* * *

Complete List of Reports and Publications

The following is a complete list of reports and publications of work done under Contract No. AF 19(604)1717 by the Division of Electromagnetic Research of the Institute of Mathematical Sciences. Lists of previous reports in this series (Nos. EM-11 through EM-97) are available upon request.

EM-98	R.A. Silverman AD 110138	On Radio Scattering by Dielectric Turbulence	Sept. '56
EM-99	B. Friedman and W.E. Williams AD 110139	Excitation of Surface Waves <u>Published</u> - Proc. J. Inst. Elec. Engrg., (London), Monograph No. 277R January, 1958.	Oct. '56
EM-100	D.S. Jones and M. Kline AD 110140	Asymptotic Expansion of Multiple Integrals and the Method of Stationary Phase <u>Published</u> - J. Math. and Phys., <u>37</u> , 1 (1958).	Oct. '56
EM-101	R.A. Silverman AD 117006	Fading of Radio Waves Scattered by Dielectric Turbulence <u>Published</u> - JAP, <u>28</u> , 506 (1957).	Jan. '57
EM-102	J.B. Keller and B. Levy AD 117007	Propagation of Electromagnetic Pulses Around the Earth <u>Published</u> - IRE, PGAP, <u>AP-6</u> , 56 (1958).	Feb. '57
EM-103	A. Shenitzer AD 117057	Diffraction Theory - Three Papers Translated from the Russian <u>Published</u> - (Abstract) - JAP, <u>28</u> , 926 (1957).	Mar. '57
EM-104	J.B. Keller AD 117078	Acoustic Torques and Forces on Disks <u>Published</u> - J. Acoust. Soc. Amer., <u>29</u> , 1085 (1957).	Apr. '57

EM-105	J. Radlow AD 117079	Diffraction of a Dipole Field by an Unidirectionally Conducting Screen	May '57
		<u>Accepted</u> - Quart. Appl. Math	
EM-106	E. Wolf AD 117277	Reciprocity Inequalities, Coherence Time and Bandwidth in Signal Analysis and Optics	June '57
		<u>Published</u> - Proc. Phys. Soc. (London): <u>71</u> , 257 (1958).	
EM-107	F.M. Ragab AD 113670	The Inverse Laplace Transform of an Exponential Function	July '57
		<u>Published</u> - Comm. Pure and Appl. Math.; <u>11</u> , No. 1, 115 (1958).	
EM-108	S.N. Karp AD 133799	Diffraction of a Plane Wave by an Unidirectionally Conducting Half-plane.	Aug. '57
		<u>Accepted</u> - <u>11</u> Nuovo Cimento	
EM-109	B. Levy and J.B. Keller AD 146784	Diffraction by a Smooth Object	Dec. '57
		<u>Accepted</u> - Comm. Pure Appl. Math.	
EM-110	J.B. Keller AD 146887	The Inverse Scattering Problem in Geometrical Optics and the Design of Reflectors	Jan. '58
		<u>Accepted</u> - PGAP	
EM-111	F. Karal and S.N. Karp AD 146888	Diffraction of a Skew Plane Electro- magnetic Wave by an Absorbing Right- Angled Wedge	Feb. '58
		<u>Accepted</u> - Comm. Pure Appl. Math.	
EM-112	E. Wolf	An Integral Representation of the Electromagnetic Field in the Image Space of an Optical System, I.	Mar. '58

- EM-114 H.E. Moses The Solution of Maxwell's Equations June '58
AD 152441 in Terms of a Spinor Notation, Part I:
The Initial Value Problem in Terms of
Field Strength and the Inverse Problem

Accepted - Phys. Rev.
- EM-115 J.B. Keller A Geometrical Theory of Diffraction July '58

Published - AMS 'Calculations of
Variations and its Appli-
cation'; (Ed., L.M.Graves)
McGraw-Hill, 1958.
- EM-116 S.N. Karp and Surface Waves on a Right Angled Aug. '58
F.C. Karal Wedge

Submitted - Comm. Pure Appl. Math.
- EM-118 J. Radlow On the Double Laplace Transforms Nov. '58
of Some Green's Functions

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- EM-119 J. B. Keller How Dark is the Shadow of a Round- Oct. '58
Ended Screen?

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- EM-120 L.B. Felsen and Relation Between a Class of Two- Jan. '59
S.N. Karp Dimensional and Three-Dimensional
Diffraction Problems
- EM-121 B. Levy Diffraction by an Elliptic Cylinder Dec. '58

Accepted - J. Math. Mech.
- EM-123 F.C. Karal Diffraction of a Plane Wave by a Jan. '59
and S.N. Karp Right Angled Wedge Which Sustains
Surface Waves on One Face

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- EM-124 S.N. Karp Vertex Excited Surface Waves on Jan. '59
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EM-125	R.A. Silverman and H.S. Shapiro	Some Spectral Properties of Weighted Random Processes	Mar. '59
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